Car Pushing Force: Traction Limiting Case

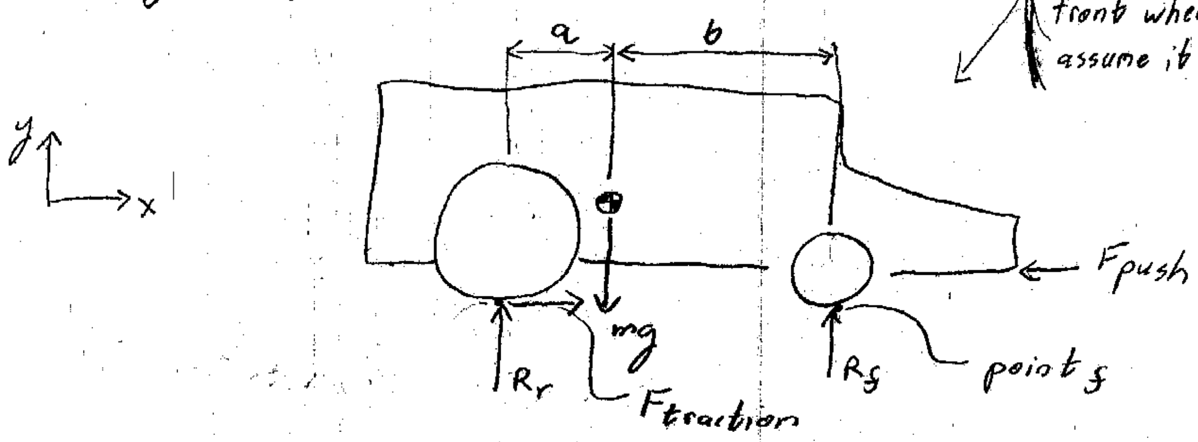
Assumptions:

- Motor is sufficiently strong so that it does not stall
- Quasi-static analysis
- Neglect internal friction in wheel bearings

Analysis Objective: Identify at what pushing force the wheels will begin to slip. This is the max pushing force for the traction limiting case:

Free Body Diagram of Car

(Note: there is no horizontal force on front wheel, since we assume it freely rotates)



$\Sigma F_x = 0$ $\quad F_{traction} - F_{push} = 0 \Rightarrow F_{push} = F_{traction}$

$\Sigma F_y = 0$ $\quad R_r + R_f - mg = 0$

$\Sigma M_f = 0$ $\quad + mg\,b - R_r(a+b) = 0 \Rightarrow R_r = \dfrac{mg\,b}{a+b}$

Maximum Possible Traction Force Occurs When:

$F_{traction,\,max} = \mu N$

$N \Rightarrow$ Normal Force

$N = R_f$

$\mu \Rightarrow$ coefficient of friction

$$F_{traction,\,max} = \mu R_r = \frac{\mu m g b}{a+b}$$

Design Guidelines.

Increase traction force by:

→ increasing μ (but there is a limit on how much μ can be increased)

→ increasing mg. (but increasing the wieght too much will increase friction in the wheels bearings and reduce pushing force)

⇒ Moving wieght over driving wheels, by increasing dimension b relative to $a+b$.
But if the dimension b is too large, the car may tip over backwards during initial acceleration

⇒ By converting car from a 2-wheel drive to a 4-wheel drive, so that traction can be generated on all 4 wheels. In this case the total maximum traction force would equal $\mu m g$.

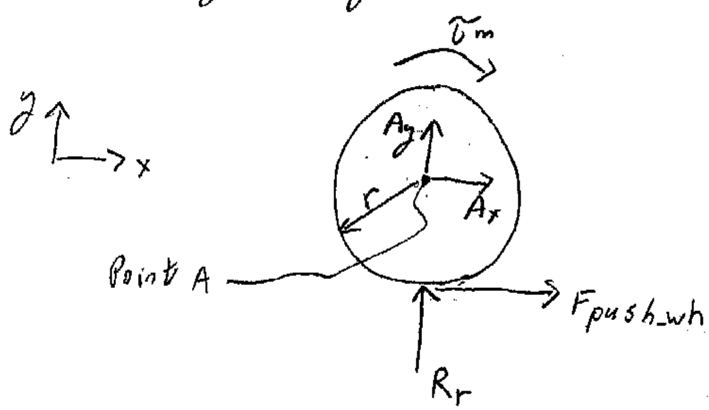
Car Pushing Force: Limited by Motor Torque

Assumptions:

- Traction is sufficiently strong so that wheels will not slip.
- Quasi-static analysis.
- Neglect internal friction in wheel bearings

Analysis Objective: Identify at what pushing force the motor will stall,

Free Body Diagram of Drive Wheel (Rear Wheel)



$A_x, A_y \Rightarrow$ Forces on axle

$\tau_m \Rightarrow$ motor torque

$R_r \Rightarrow$ reaction at rear wheel

$r \Rightarrow$ radius of wheel

$F_{push\_wh} \Rightarrow$ pushing force per drive wheel (there are two)

$\Sigma F_x = 0 \qquad F_{push\_wh} + A_x = 0$

$\Sigma F_y = 0 \qquad R_r + A_y = 0$

$\Sigma M_A = 0 \qquad F_{push\_wh} \cdot r - \tau_m = 0$

$$F_{push\_wh} = \frac{\tau_m}{r}$$

Maximum Pushing Force (assuming traction is not lost) occurs at stall torque, i.e. when

$$\tau_m = \tau_{stall}$$

Since there are motors on the right and left side of the car:

$$F_{push,\,max} = \frac{2\,\tau_{stall}}{r}$$

Design Guidelines

*) Decrease r to increase pushing force, but this will also reduce maximum car speed.

*) One could put a gear reduction between the motor and the wheel to increase pushing force. This will also reduce car speed.

Combining Results from Traction Case and Stall Torque Case

Analytical Calculation.

*) The lowest value of pushing force calculated from both cases will be the actual pushing force. If the traction pushing force is lower, than the wheels will begin to slip before the motor stalls, and vica versa.

Experimental Verefication.

*) Drive car into wall and observe if wheels slip or motor stalls. You can then have car pull against spring scale to measure pushing force.