

Tutorial of Point Mass Pendulum Analysis

Problem Description

There is a [model clock](#) that is composed of an escapement wheel and a swinging pendulum. The timing of the clock depends on the natural frequency of the pendulum, and the escapement wheel provides energy to overcome frictional losses and keep the pendulum oscillating.

Objective

To find the natural frequency of the pendulum using data obtained from AutoCAD and actual measurements. In this analysis we assume that all the mass of the pendulum is concentrated at a single point, and thus the pendulum has no rotational inertia. This analysis is simple to perform and useful for verification purposes. See the tutorial for the [Physical Body Pendulum Analysis](#) for a more accurate analysis of the pendulum.

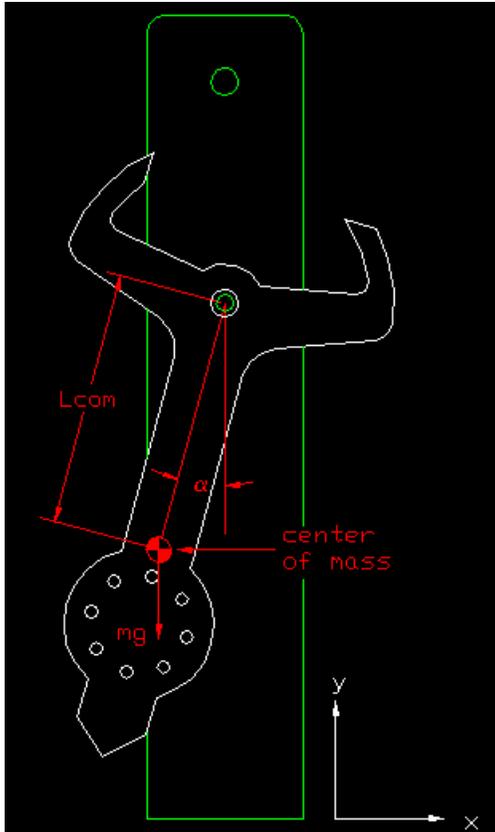
Assumptions

1. Friction can be neglected.
2. The maximum angle of motion α , is relatively small.
3. The mass of the pendulum is concentrated at one point.
4. The pendulum swings freely. (we do not consider the effect of the escapement wheel)

Analysis

Free Body Diagram

[return to top](#)

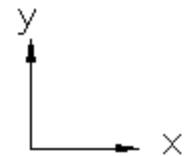
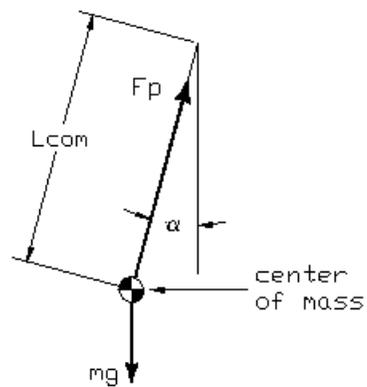


F_p = tension force in pendulum

L_{com} = effective length of pendulum

mg = gravitational force on pendulum

α = angle between F_p and mg



Basic Equations of Point Mass Pendulum

natural frequency in radian $\omega = \sqrt{\frac{g}{L_{com}}} \left(\frac{\text{radians}}{\text{sec}} \right)$

[\(click for a derivation of the point mass pendulum frequency\)](#)

g = gravity

L_{com} = effective length of pendulum (distance from pivot to the center of mass of the pendulum)

Calculating the Center of Mass of the Pendulum

The Center of Mass Analysis can be broken up into two parts. The first part is without the bolts in the pendulum, and the second part includes the weight of the bolts. An intermediate verification should be performed after each analysis.

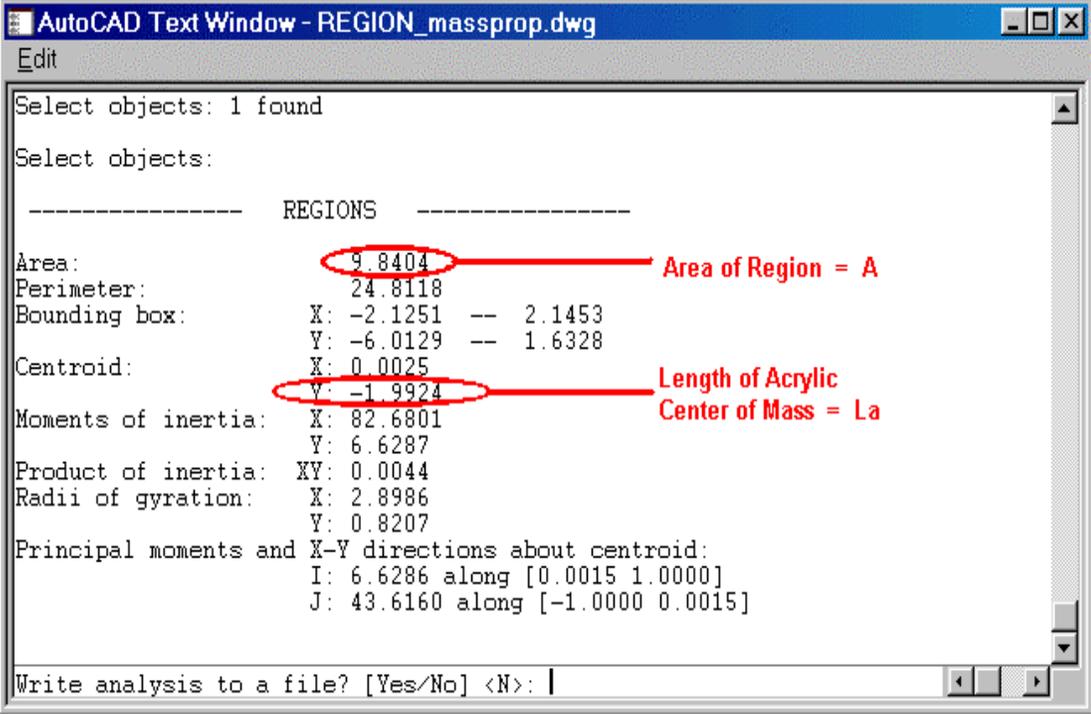
Finding the Center of Mass of the Pendulum Without Bolts [return to top](#)

You can use the Mass Properties Tool in AutoCAD to calculate the center of mass of your acrylic pendulum. First you must convert your pendulum outline into a Region, then AutoCAD can calculate the mass properties.

Note: All lines in the pendulum must be connected in order for AutoCAD to be able to create a Region.

1. Command: **EXPLODE** (then select the outline of the pendulum).
2. Command: **REGION** (then select the outline of the pendulum).
3. Pull down **TOOLS** and select **MOVE UCS** (then select center of pendulum pivot).
4. Pull down **TOOLS** and select **INQUIRY** then **MASS PROPERTIES** (then select the pendulum).

AutoCAD should return the window shown below. If you are having problems making a region AutoCAD, click on the link. [Troubleshooting: Making a Region](#)



The screenshot shows the 'AutoCAD Text Window - REGION_massprop.dwg' with the following text:

```
AutoCAD Text Window - REGION_massprop.dwg
Edit
Select objects: 1 found
Select objects:
----- REGIONS -----
Area: 9.8404
Perimeter: 24.8118
Bounding box: X: -2.1251 -- 2.1453
               Y: -6.0129 -- 1.6328
Centroid: X: 0.0025
           Y: -1.9924
Moments of inertia: X: 82.6801
                   Y: 6.6287
Product of inertia: XY: 0.0044
Radii of gyration: X: 2.8986
                  Y: 0.8207
Principal moments and X-Y directions about centroid:
                  I: 6.6286 along [0.0015 1.0000]
                  J: 43.6160 along [-1.0000 0.0015]
Write analysis to a file? [Yes/No] <N>: |
```

Red circles highlight the values 9.8404 and -1.9924. Red arrows point from these values to the text 'Area of Region = A' and 'Length of Acrylic Center of Mass = La' respectively.

Data

Record the Area and Centroid of your pendulum from AutoCAD. The Centroid given is the length from the pivot of the pendulum to its center of mass. Using the [density](#) you can calculate the expected weight of the acrylic.

Note: The acrylic is nominally 0.25" thick, but it varies from batch to batch. Use a caliper for a more accurate measurement.

Volume of Acrylic $V = A \cdot t$ (inches³) (A = area, t = thickness)

Weight of Acrylic $W_a = \rho \cdot V$ (oz) (ρ = density, V = volume)

Intermediate Analysis and Verification

Where ever possible, one should verify theoretical results by comparing them to physical properties and discussing where the discrepancies may come from.

* Verify the pendulum weight without the bolts using the scale.

* Verify the pendulum center of mass without the bolts by balancing the pendulum on ones finger.

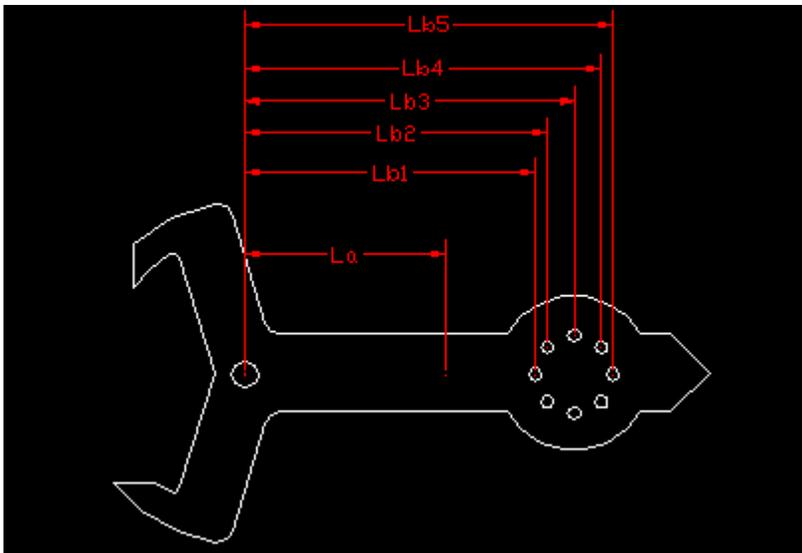
If you find unacceptable large discrepancies, then find the source of the error and correct it.

Option: Do you want higher accuracy? If so, click [here!](#)

Finding the Center of Mass of the Pendulum with Bolts [return to top](#)

A key variable needed in the calculation of the center of mass of the pendulum with bolts is the distance of every bolt for the pivot. You can use AutoCAD's Linear Dimensioning to accomplish this with ease.

1. Have an AutoCAD drawing ready to be measured.
2. Pull down the **DIMENSIONS** menu from the top of the screen.
3. Select the **LINEAR** option, and then connect the two regions that you want to find the distance between.
4. Press the space bar to repeat the process.
5. One can also select the **VIEW** menu, and then select the **TOOLBARS** option. Check marking the dimensions box will allow for an easy dimension measurements every time
6. Record the various distances as shown below.



Data

Record the various lengths of the bolts to the pivot. The reason we are only taking the vertical values of the distance is because the bolts on the pendulum are located in a symmetrical manne. Thus, we consider that the pendulum's center of mass is already located somewhere along the central line.

$$\text{total weight of pendulum } W_t \text{ (oz)} = W_a + n \cdot W_b \quad (W_a = \text{weight of acrylic} \\ n = \text{number of bolts} \\ W_b = \text{weight of bolt})$$

effective length of center of mass

$$L_{\text{com}} \text{ (inches)} = \frac{W_a \cdot L_a + W_b \cdot (L_{b1} + L_{b2} + L_{b3} + \dots + L_{bn})}{W_t}$$

Intermediate Analysis and Verification

- * Verify the pendulum weight with the bolts using the scale.
 - * Verify the pendulum center of mass with the bolts by balancing the pendulum on ones finger.
- If you find unacceptable large discrepancies, then find the source of the error and correct it.

Point Mass Pendulum Frequency Calculation [return to top](#)

Use the values you have obtained to solve for the frequency of the pendulum, and the calculated tota time that the clock should run.

$$\text{natural frequency in radian } \omega = \sqrt{\frac{g}{L_{\text{com}}}} \quad \left(\frac{\text{radians}}{\text{sec}} \right)$$

$$\text{natural frequency in Hertz } f = \omega \left(\frac{\text{rad}}{\text{sec}} \right) \cdot \left(\frac{\text{cycles}}{2\pi \text{rad}} \right) = \frac{\omega}{2\pi} \left(\frac{\text{cycles}}{\text{sec}} \right) \quad \{\text{Hz}\}$$

$$\text{period of oscillation } T = \frac{1}{f} \quad \left(\frac{\text{sec}}{\text{cycle}} \right)$$

The total time that the clock will run for depends on three things.

1. The period of the pendulum which is the amount of time that it takes for the pendulum to do one oscillation or cycle.
 2. The number of cycles the pendulum can do per rotation of the escapement wheel.
- What this basically boils down to is the number of teeth on the escapement wheel.
3. The number of rotations the escapement wheel performs.

$$\text{total time} = \frac{\text{sec}}{\text{cycle}} \cdot \frac{\text{cycles}}{\text{revolution}} \cdot \text{revolutions}$$

Comparison of Calculated Time to Actual Time

What were the results? Compare the results to the actual time the pendulum runs and see if there is a discrepancy. If so, why?